

## Oil Production Test Study

The June 2005 issue of the journal *Decision Analysis*<sup>1</sup> (Volume 2, Number 2) contained three papers largely devoted to discussing different methods of finding a strategy for and valuing an oil production project with options.<sup>2</sup>

We'll begin by briefly describing the oil production project, the method used by Brandao et al., and how Smith's method improved the analysis. Then we'll discuss how the problem is modeled using the Provisdom Decision Platform to match Smith's result and how the problem can easily be made more accurate and extended to handle other information like scenarios with Market relationships.

## Oil Production Project

Brandao et al. described a corporation in the petroleum industry that has a decision to invest \$180 million in oil production at a site with an estimated 90 million barrels of remaining reserves. The corporation will take over a 75% interest, with another partner owning the remaining 25%. The project length is taken to be 10 years, with an initial production level of 9 million barrels per year, declining at a rate of 15% yearly. After 5 years, the corporation has the option to continue production, buy out the partner for \$40 million, or divest at a price of \$100 million. The two uncertainties are variable operating expenses (VOPEX) and the price of oil, both modeled as geometric Brownian motions. The VOPEX is assigned an initial value of \$10/barrel with a growth rate of 2% and volatility of 10%, while the oil price has an initial value of \$25/barrel with a growth rate of 3% and volatility of 15%.

Brandao et al. first valued the project without options by calculating the expected cash flows and discounting them at the company's weighted average cost of capital (WACC). Next they simulated the cash flows with the two uncertainties to arrive at the project's volatility. Finally, treating the project without options as an underlying security, they priced the project with options by building a decision tree using a risk-neutral approach. Brandao et al. advocated the use of a decision tree to solve the problem due to the availability of *Decision Analysis* software which utilized this representation, and the ability to include *scenarios* (uncertainties with discrete outcomes).

Smith improved several of the technical and fundamental elements of this analysis. First, Smith realized that by reducing a multivariate model to a univariate one, information is almost always lost. In this problem, transparency is sacrificed and Smith shows how the project's volatility is over-estimated. Second, Smith advocates a fully risk-neutral solution instead of using the WACC, as the latter is a poor approximation. And third, Smith argued that using a decision lattice instead of a tree would be better for this problem due to advantages in computational efficiency.

The Provisdom Decision Platform provides further improvements:

---

<sup>1</sup> <http://www.informs.org/site/DA/>

<sup>2</sup> The original paper by Brandao et al. can be found at [http://faculty.fuqua.duke.edu/~jes9/bio/BDH\\_Article.pdf](http://faculty.fuqua.duke.edu/~jes9/bio/BDH_Article.pdf). James Smith's commentary can be found at [http://faculty.fuqua.duke.edu/~jes9/bio/BDH\\_Comment.pdf](http://faculty.fuqua.duke.edu/~jes9/bio/BDH_Comment.pdf). The reply of Brandao et al. can be found at [http://faculty.fuqua.duke.edu/~jes9/bio/BDH\\_Reply.pdf](http://faculty.fuqua.duke.edu/~jes9/bio/BDH_Reply.pdf).

- Ease of model creation due to the ability to provide information in a familiar form and high level of abstraction from the underlying solution engine.
- Computational efficiency obtained by joining different paths when they reach the same state. Problems which naturally fit a lattice will be solved in lattice, while those requiring a full tree will utilize the tree representation. Many realistic problems live somewhere between these two extremes.
- Maximization of shareholder value even when non-assets with a Market relationship are included (i.e. a fully risk-neutral solution is obtained given the supplied information).

## Building the Oil Production Problem

The oil production problem can be represented by five Information Rules:

1. Resolve the VOPEX uncertainty each period
2. Resolve the oil price uncertainty each period
3. Generate the payoff from oil production each period and update the oil production level (which was modeled deterministically in the original papers).
4. When we've reached Year 5, choose between the Buyout, Continue, or Divest options; include payoffs where appropriate.
5. When we've reached Year 10, or if we choose to divest, end the model.

There are four state variables:

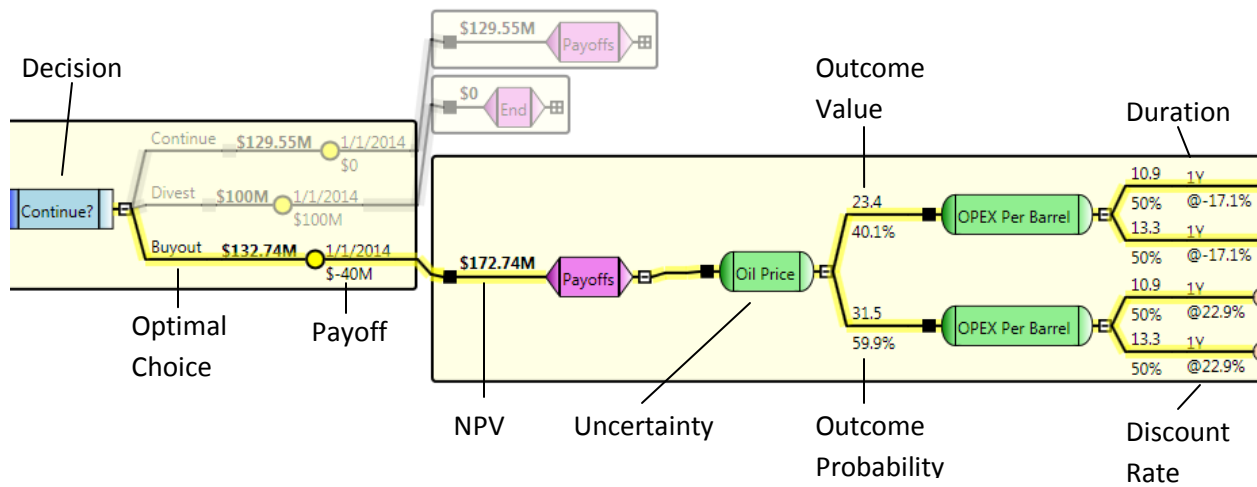
1. Time
2. Price of oil
3. Current VOPEX
4. Current decision to divest, buy-out, or continue.

To match Smith's solution, we:

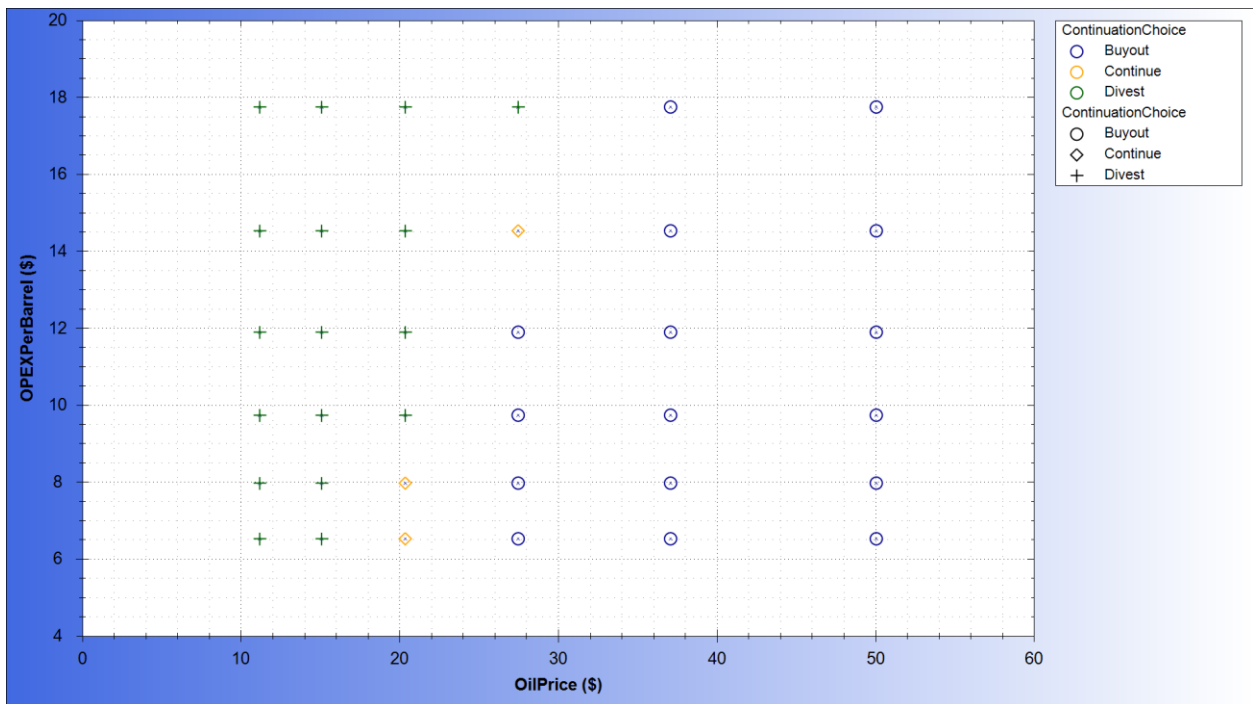
- Translated the parameters to continuous rates for consistency and for flexibility in setting period lengths.
- Set the yield on the oil price equal to the risk-free rate to get Smith's oil price risk-neutral growth rate of 0%.
- Modeled oil production discretely, having a constant value over the course of the year and then suddenly decreasing it at the end of each year by 15%.

Smith's fully risk-neutral solution gives a value of \$421M. The Provisdom Decision Platform's solution using the same information is \$421.34M.

A portion of the solution is shown below.



To see the Year 5 strategy, we can query the solution database, and then plot the strategy against the relevant variables. This plot is shown below.



The points above correspond to the discretized values of oil price and VOPEX. To get the optimal strategy for specific values of oil price and VOPEX, we simply begin the model at Year 5 with the requested values. For example, if the oil price is \$43 and the VOPEX is \$17, then the optimal strategy is to buy out the partner.

## Missing Information

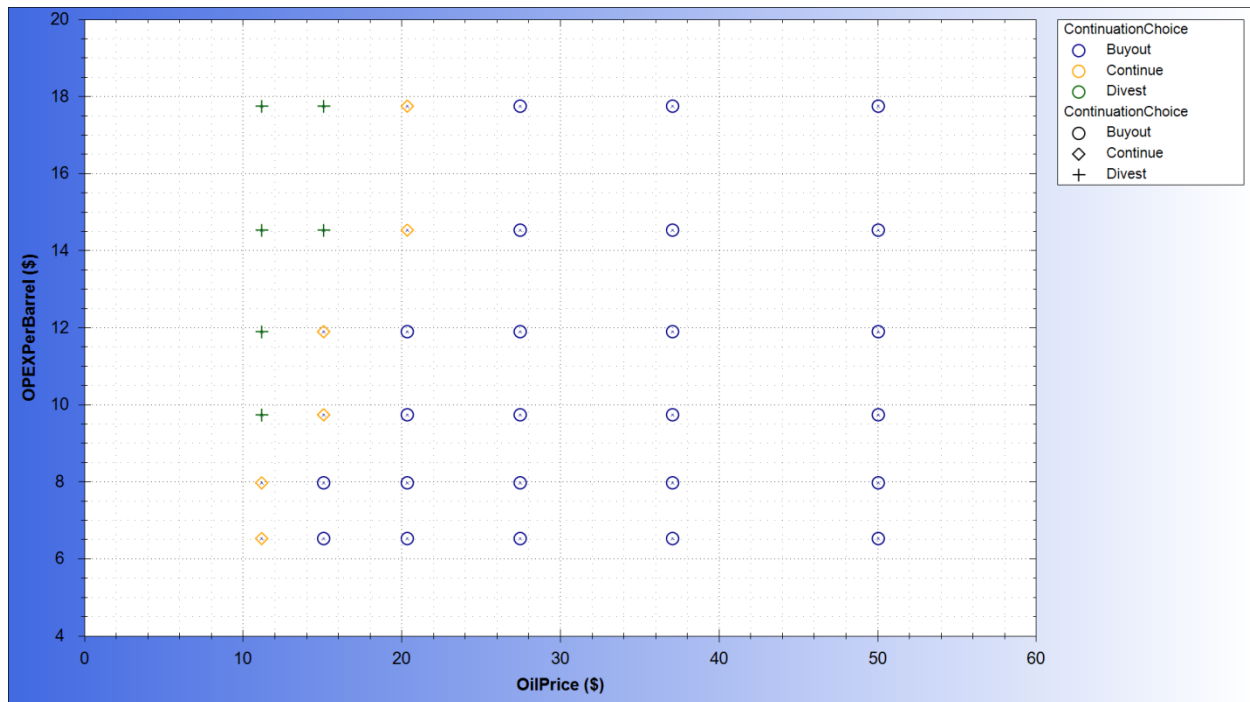
Brandao et al. state that the VOPEX may have a relationship with the Market, yet didn't specify one in their analysis; correspondingly, Smith did not include this relationship either. Increasing the Market correlation with VOPEX from 0% to 20% increases the NPV from \$421.34M to \$429.55M, changes the optimal strategy, and shows the potential importance of any available information regarding the Market relationship.

Since Brandao et al. didn't directly model the oil price in a risk-neutral framework, Smith was free to decide to model the oil price as having a 0% risk-neutral growth rate. This is equivalent to setting the yield of oil equal to the risk-free rate. A yield of 3% gives an NPV of \$475.92M, while a yield of 7% gives \$372.07M. Clearly, including any information we have regarding the yield may be important.

## More Accurate Modeling of Available Information

To more accurately model the available information, we can model oil production as a continuous exponential process, as opposed to the discrete exponential process used in the original papers, and we can model the fixed OPEX costs as a continuous stream. These changes lower the NPV from \$421.34M to \$403.7M and highlight the importance of accurately modeling the given information.

Since only being allowed to buy-out or divest at Year 5 seems unrealistic we added the option to divest every year and the option to buy-out every year from Year 5 onward due to a contractual constraint (if the buy-out is allowed from the beginning, the buy-out will be chosen immediately). We accomplish this by simply removing the condition that the decision occurs only at Year 5 and by adding a "last decision time" state variable to ensure the decision is made only once per time period. The Year 5 strategy with decision flexibility is shown below:



Not surprisingly, given the option to buy out at later times, we choose to divest less often than in the original case. The additional flexibility increases the shareholder value of the project from \$403.7M to \$450.06M.

### **Model Extension -- Political Unrest**

We'd also like to explore how the optimal strategy would change if the oil reserves exist in a country with potential political instability. We'll model the political instability as a scenario with two possible outcomes, "Good" and "Unrest." If "Unrest" occurs, we believe the production facility will be lost (project terminates with value \$0). Our best information tells us that the probability of "Unrest" is 5% per year and that if "Unrest" occurs over the year, on average the Market will have a growth rate, standardized by its volatility, which is 10% less than currently expected over that year. Since the oil price is an asset and therefore has an implicit relationship with the Market, the oil price also has an implicit relationship with the political instability. This relationship is made explicit and transparent when viewing the model's conditional probabilities; for example, in Year 1 the conditional probability of "Unrest" given that oil went up is 5.27%, and when conditioned on oil going down is 4.82%. The possibility of political unrest lowers the NPV from \$450.06M to \$375.64M.

### **Conclusion**

Modeling this oil exploration problem with the Provisdom Decision Platform simply required creating 5 Information Rules, setting 4 state variables, and elucidating the model properties. Due to the frequent joining of states in the model, even with yearly decision flexibility, it took only a couple seconds to run the model on an off-the-shelf 2007 laptop. All Market information was accurately modeled; like the oil price yield, the Market correlation with VOPEX, and the Market relationship with the political unrest situation. Outputs like natural probabilities and discount rates are terms that managers are likely to find more familiar than constructs such as risk-neutral growth rates and risk-neutral probabilities.