DRAM Supply Chain Test Case

Story

A Vice President (the VP) of a major internet information services company (the Company) was charged with all DRAM purchasing to support datacenter operations. The current strategy was to simply purchase 50K units per week, the nominally required amount, thus avoiding inventory costs.

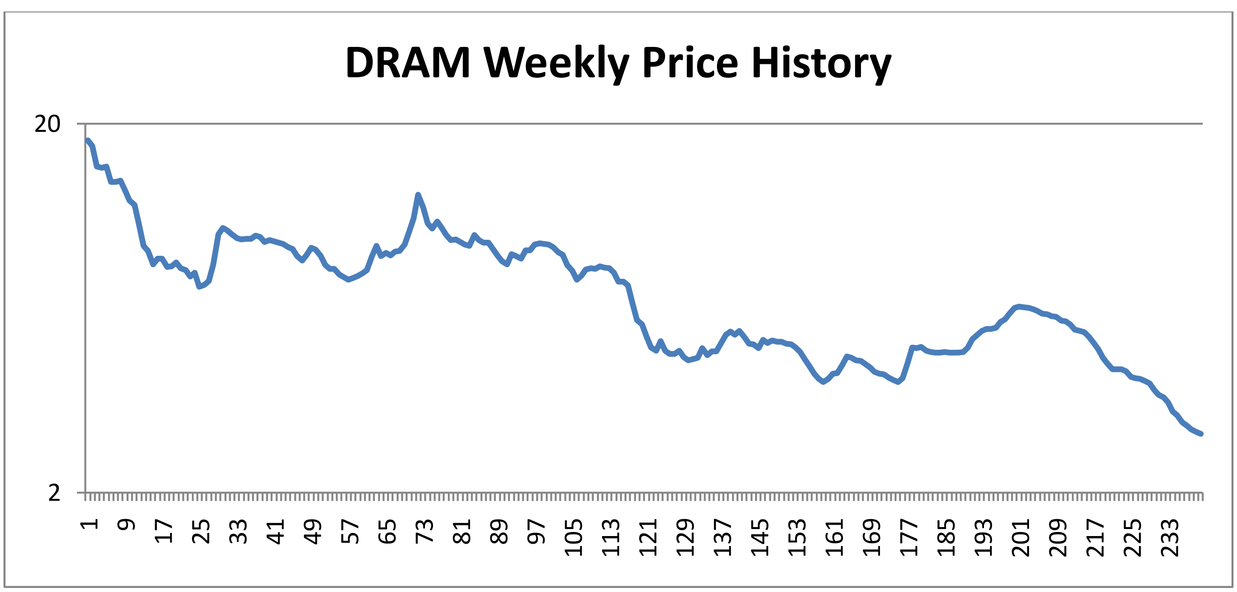
The VP recognized that his strategy lacked agility in the face of fluctuating DRAM prices and the potential of supply shortages.

The VP asked a top manager/analyst (the Analyst) to provide an improved strategy. Using Provisdom's help, the Analyst rapidly incorporated data on the price history of DRAM along with corporate information about possible supply shortages to develop a flexible strategy.

Business Problem

The Analyst had the following relevant information regarding DRAM.

- Each week a purchasing decision will be made regarding how much DRAM to purchase.
- On average, 50K new DRAM will be needed each week.
- The cost of putting DRAM into inventory will be $.50 per unit per year.
- Each week there is a 5% chance that a DRAM supply shock will occur, resulting in no available DRAM for the following week.
- The cost of not meeting DRAM need will be $500 per unit per year.
- The above information will be valid for 3 years.
- Weekly DRAM price data from the past 241 weeks is shown below.



The Analyst's Model

The Analyst hadn't built a complete model, but he had analyzed the past DRAM prices, chosen to make several assumptions, and determined a method for comparing various proposed strategies.

Future DRAM price

The Analyst had decided to model the past DRAM price as a lognormal mean-reverting process with a constant volatility that tended toward a target price with a fixed growth rate.

He solved for the parameters using the logarithm of just the first 200 data points and the following process.

a) Solve regression model for initial target price and slope (constant growth rate).
b) Solve least-squares problem to find regression rate of data (speed at which actual price tended toward calculated target price). This problem was solved assuming that the growth rate should be proportional to the difference between the actual price and the target price without consideration for the time between data points.
c) Solve least-squares problem to find volatility of past data using calculated target prices and regression rates.

Assumptions

1. Future DRAM price will follow parameters from a mean-reverting process calculated with the method outlined above.
2. Ignore possible supply shock effects
3. Exactly 50K DRAM units would be needed per week.

Comparing Strategies

To compare strategies, the Analyst proposed to test various strategies against the last 41 past data points, i.e. starting from data point 201. The strategy that had the lowest total cost would be determined to be the best (no discounting was to be applied).

Provisdom's First-Cut Model

Using the Provisdom Decision Platform, Provisdom proceeded to create a general model and match the Analyst's DRAM price analysis. Therefore, we used the same procedure as the Analyst to find the target prices, the regression rate, and the volatility of the past DRAM prices.

To solve for the strategy that maximized shareholder value, Provisdom then made the future DRAM prices follow this process over 8 weeks of detailed modeling, then used an approximated payoff (due to limited computing resources) for the time between 8 weeks and the end of the model. The approximated payoff considered the amount of DRAM in inventory, the actual price, the target price and slope, and the amount of DRAM needed per week. We also solved for the past correlation between the DRAM price and the Market using weekly S&P data to determine the proper discount rates to maximize shareholder value.

Results

The optimal strategy was found to be to simply buy 50K each week as the VP had been doing all along. Of course, this model was over-simplified and required improvement.

Improving the DRAM Model

Future DRAM price

To begin to address Assumption #1 above (future DRAM price model), we still assumed that the past DRAM prices followed a lognormal mean-reverting process with a constant volatility that tended toward a target price with a fixed growth rate, but we used the following more accurate method and used all 241 past data points to determine the future parameters of the DRAM prices.

a) Solve least-squares problem to simultaneously find initial target price, slope, and the regression rate of the data. This time we integrated the growth rate over the time between data points (removing about 98% of the approximation error).
b) Solve least-squares problem to find volatility and Market correlation of past data using calculated target prices and regression rates.

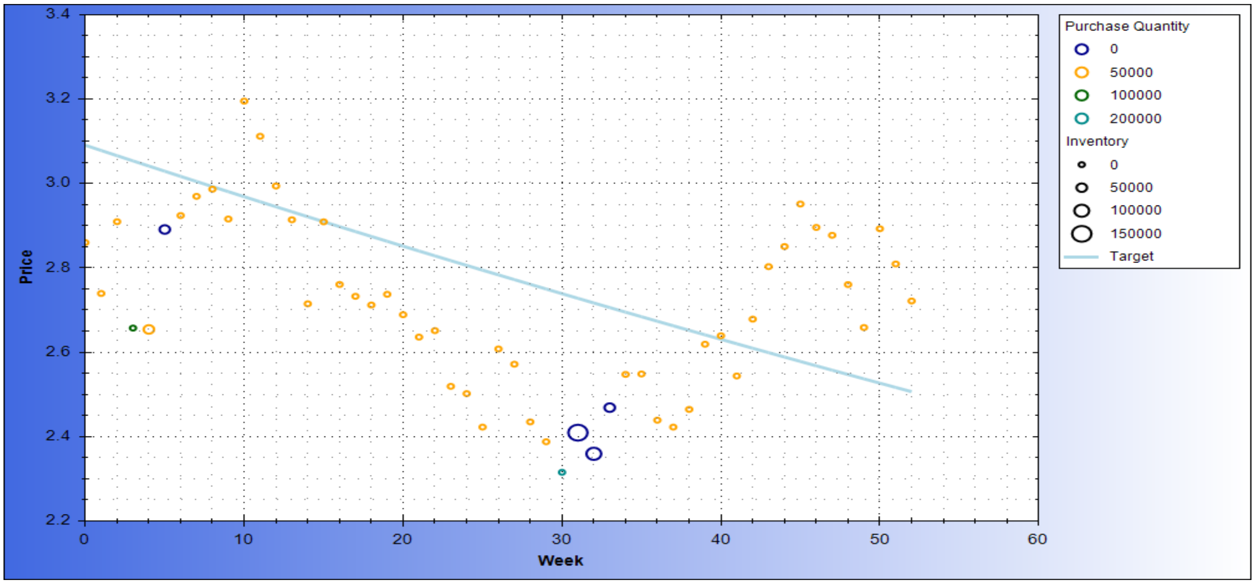
Since the DRAM price had dropped dramatically over the last 41 weeks, using the entire data set drastically changed the calculated parameters. The price had recently dropped rapidly enough to fall below the calculated target line and change the optimal strategy.

Optimal Strategy

With an improved model for the DRAM future price, the optimal strategy changes dramatically. The initial strategy is now as follows:

- Begin by buying 50K for each of the first two weeks.
- If the DRAM price drops to around $2.65 at that time, buy 100K.
- If the price continues to drop to $2.56 the following week, buy 200K more.
- If it instead rises to $2.77, buy none.

In practice, the prices are likely to not fall exactly on the discretized prices used in the model. Instead, the updated values are entered into the model and it is re-solved. Below is a graph of a sample simulation run over 52 weeks using this model (modeled with 7 weeks of details instead of 8).



The blue line represents the target DRAM price. Notice how the large purchases are made when the price is well below that line. With no supply shocks to consider, the inventory is generally kept low.

More Accurate Model

Supply Shock

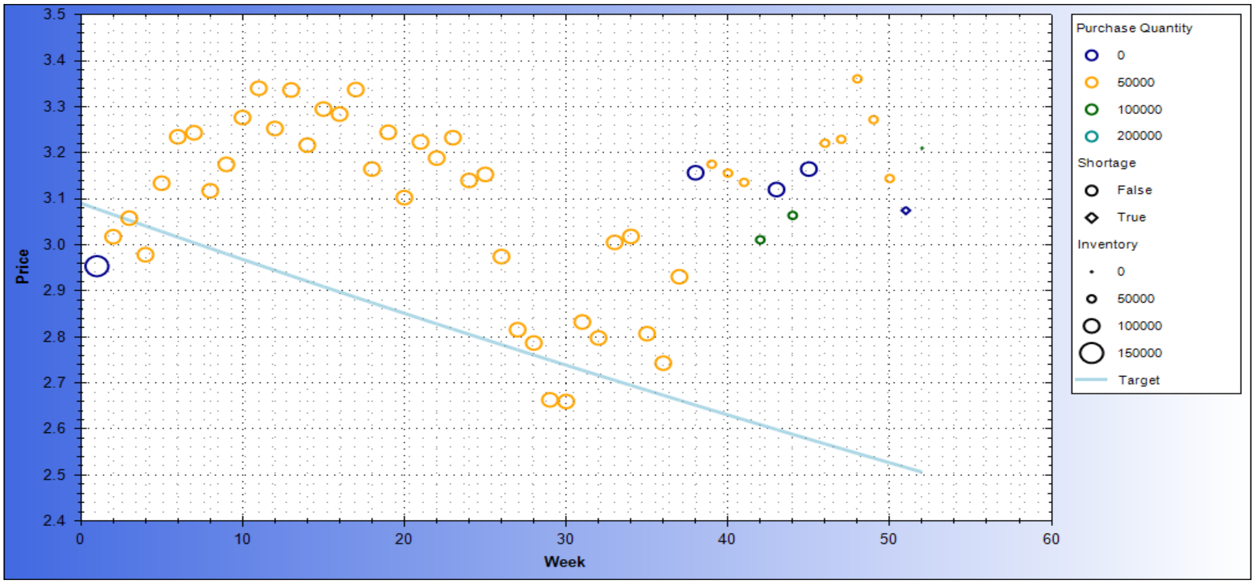
To remove Assumption #2 (supply shocks), we modeled the chance of a supply shock occurring each week and the associated cost for possibly not meeting the needed DRAM demand.

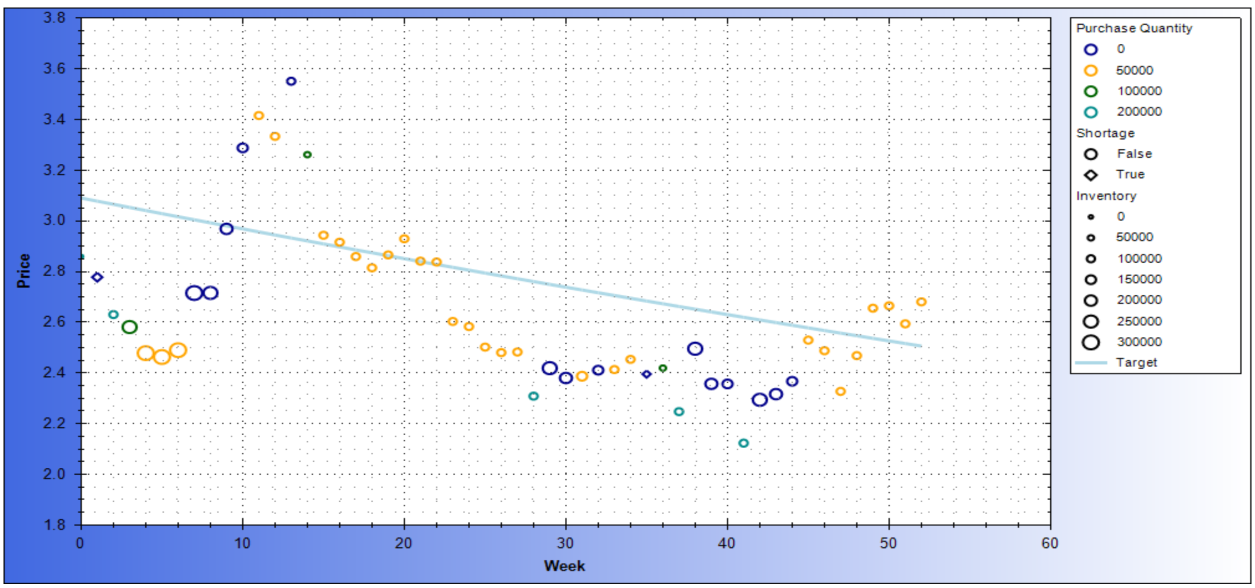
Optimal Strategy

With a more accurate model, the optimal strategy becomes:

- Load up on 200K of DRAM immediately to avoid the potential cost associated with not meeting demand if there is a supply shock.
- After the first week, if the DRAM price drops, keep the inventory level at 200K.
- If the price drops again, raise the inventory level to 250K the following week.
- If the price rises, let the inventory level drop to 150K.
- If the price rose in the first week, then put the inventory level at 150K for any price between $2.87 and $3.11.

The large difference in strategies between models shows the importance of getting all of the most relevant information into decision models. Two sample simulations are shown below (modeled with 7-week details).





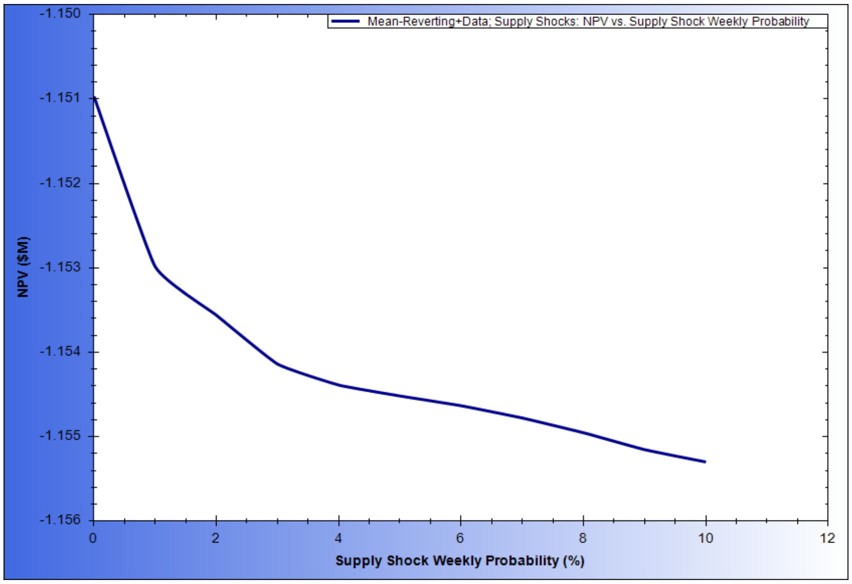
In both of these simulations, notice that when the price gets way above the target line, the inventory is held in the 50K-100K range, but when the price drops well below the target line, the inventory is in the 200K-300K range.

Shareholder Value

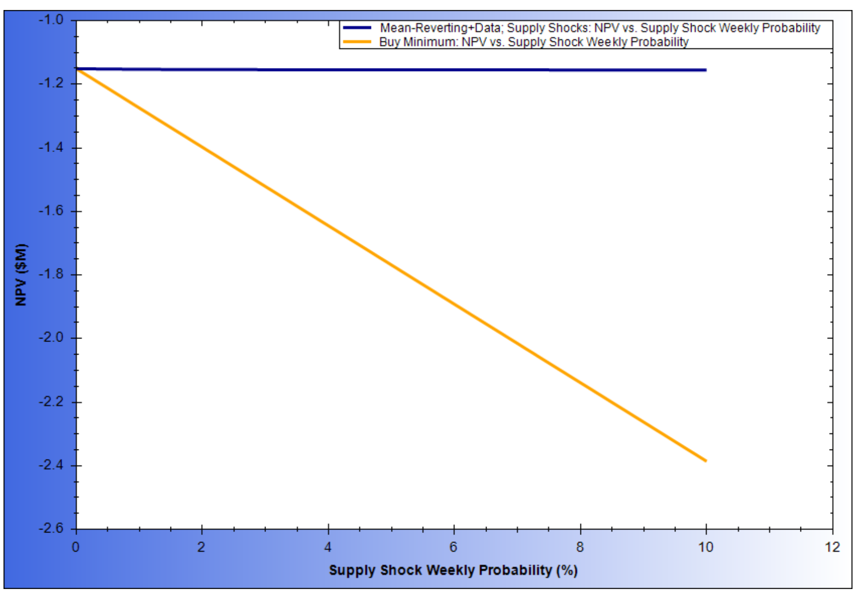
The resulting model indicated the optimal amount of DRAM to purchase given the current price and observed market conditions. This new conditional seven-week strategy had a shareholder value of -$1.15M. The Company's current strategy and strategy found in the first-cut model of buying 50K DRAM every week had a shareholder value of -$1.77M. This means that the total cost of shortages and buying and storing DRAM in terms of shareholder value decreased by over 35% with the new strategy.

Feedback and Analysis

To give the VP an idea of how the chances of supply shocks affect the shareholder value, we created a graph of the NPV against the weekly probability of a supply shock (shown below).



An informed strategy allows us to preserve shareholder value in the face of different supply shock rates. However, the original strategy with no flexibility can result in great damage. The graph below illustrates this, with the blue line representing the graph above, and the orange line showing shareholder value as a function of supply shock rate when we use the fixed 50K/week strategy:



Notice how the blue line (optimal strategy NPV) looks nearly flat compared to the orange line (50K per week).

Other Models

To further address Assumption #1 (mean-reverting DRAM prices), we also built models that assumed that the DRAM prices followed:

- A constant growth rate.
- A cyclical process.
- A constant, but unknown growth rate that would be learned about over time.

Each of these models actually seemed to fit the past data more closely than the mean-reverting process. See Appendix for more on these models. The most precise way to address the DRAM prices would be to begin with any information the Company had about DRAM prices before the data. For example, they believed there was a chance that DRAM prices would be well-modeled by a mean-reversion process, but they couldn't have known that for sure. We could model their uncertainty and then use the data to update the Company's beliefs about the various processes the DRAM prices may follow.

In addition, as the DRAM prices unfold throughout the model, the parameters should be updated at each step. For example with mean-reversion, if the DRAM price starts rising above the target line, the target estimate should rise slightly as well. The uncertainty in our target line should be included in the overall uncertainty of DRAM prices, as should the uncertainty in the process (i.e., mean-reversion, cyclical, etc.).

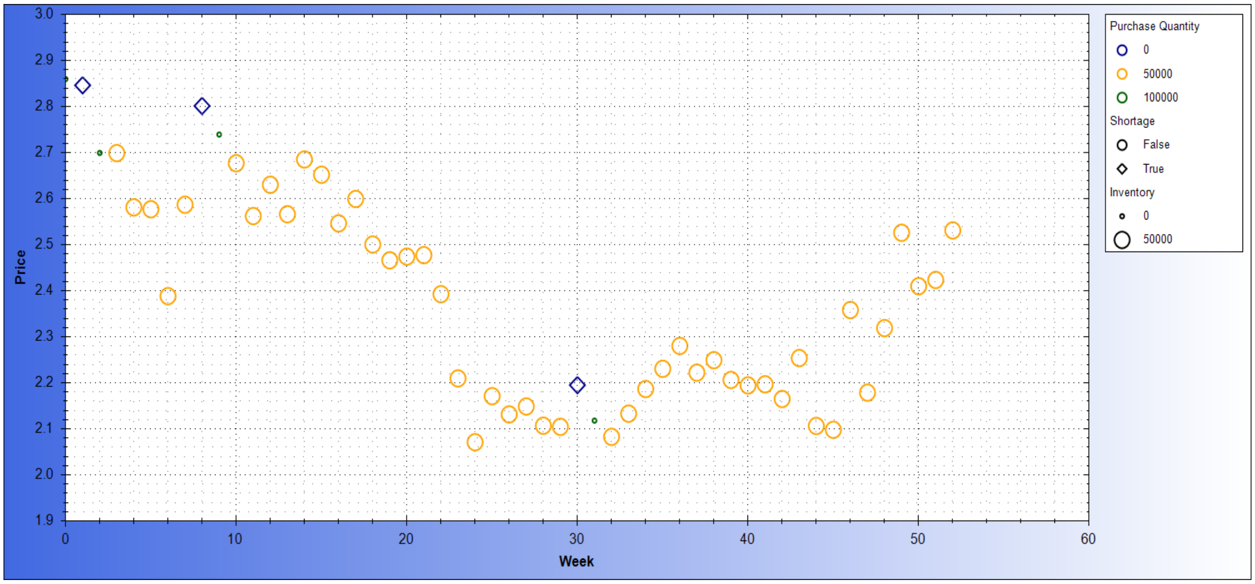
To address Assumption #3 (future weekly need is constant), we built one model with varying but known weekly demand and one model with unknown demand (it could be 20K or 80K with equal chance).

Appendix

For DRAM prices following a process whose discretization recombines, like the constant growth rate process and the cyclical process, we are able to model more weeks in detail.

Constant Growth Rate

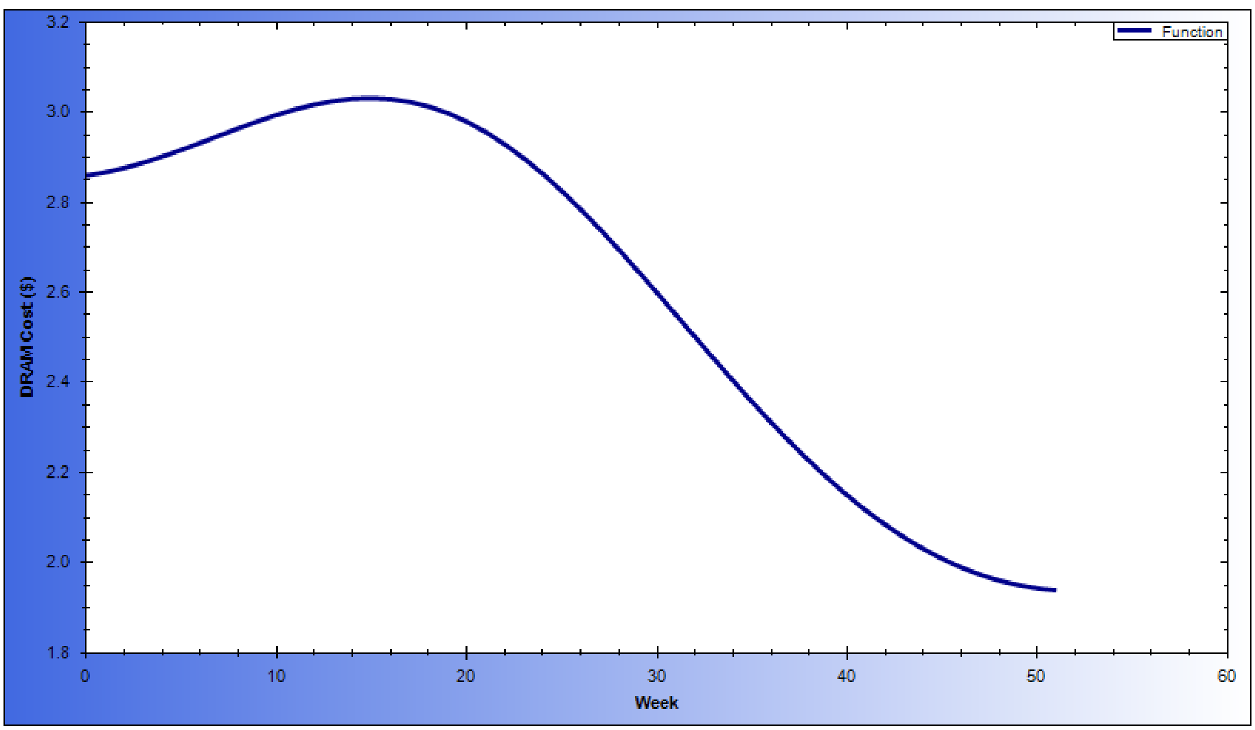
The chart below shows the output from a simulation of a 14-week model.



With a constant growth rate for DRAM, the optimal strategy is simply to hold 50K in inventory.

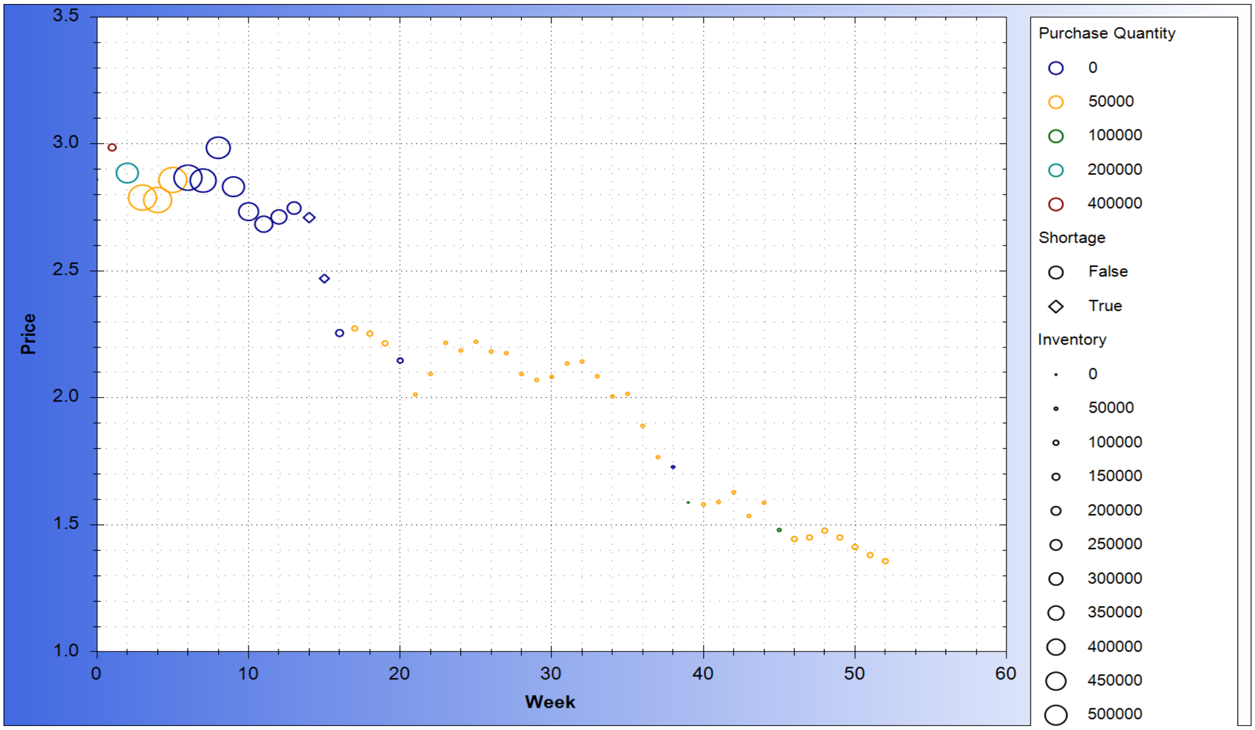
Cyclical Process

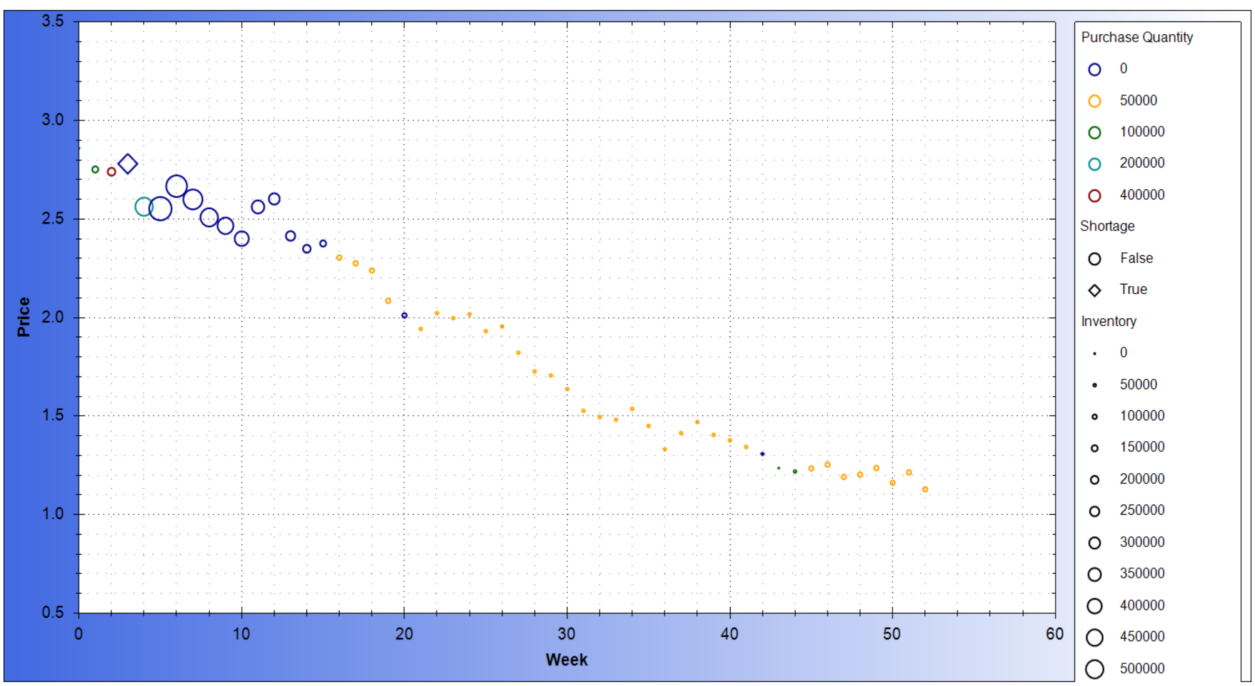
Below is a graph of the most typical cyclical motion.



The cycle is just over a year long and begins around week 7. The cycle effect is about twice as strong as the general downward trend. So, we'd expect the price to peak a little before week 20 and bottom out a little after week 47. From the graph, we can see it peaks around week 16 and is bottoming out around week 52.

Below are two charts of simulations of this cyclical motion in a 14-week model.





Notice that the larger purchases are made around weeks 42-45 regardless of the price because that's about the time we expect the price to start bottoming out. The other large purchases are made at the very beginning of the model in preparation for a possible supply shock and an early rise in price. In both simulations, no purchase was made at week 20 since that is about the time we expect the price to start dropping rapidly.